Unit 1 Test Guide

1.1 - 1.4 are covered in such as they are "assumed knowledge" for an Honors PreCalculus course and those skills may be called upon as part of a larger problem

1.5 - 1.11 - The Majority of the Test

Functions: Definition. ways of expressing. vertical line test. function notation.
- parent functions and their graphs
- basic analysis (increasing, decreasing, domain, range, continuity, etc)
- piecewise functions

Graphs of functions.
- Symmetry:
- Odd and Even Functions, proving in different forms
- Transformations

Combinations of functions:
- Adding, subtracting, multiplying, dividing
- Composite functions – functions of functions

Inverse functions.
- How to find. horizontal line test, domain and range issues.

1.12 - Average Rate of Change
- interpreting positive, negative, zero
- secant lines

Resources for Review
- Class notes and classwork (some keys are on the wikispace)
- Nightly homework (keys are stored in the back of the classroom)
- Warm-ups
- Quizzes
    - it is strongly recommended that you review any errors from a quiz and make corrections
- Practice Packet attached
    - You are not expected to do every problem in this packet!
    - Pick and choose what you need to do to be prepared
    - There is no minimum number of questions that you must complete
    - Keys will be posted to the wikispace over the weekend and available in class

Warning | This unit test will not include a calculator section

CHAPTER 1 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–10, match the graph with the corresponding function (a)–(j) from the list below. Use your knowledge of function behavior, *not* your grapher.

(a) $f(x) = x^2 - 1$ (b) $f(x) = x^2 + 1$

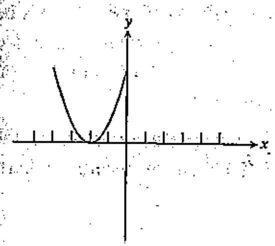
(c) $f(x) = (x - 2)^2$ (d) $f(x) = (x + 2)^2$

(e) $f(x) = \dfrac{x - 1}{2}$ (f) $f(x) = |x - 2|$

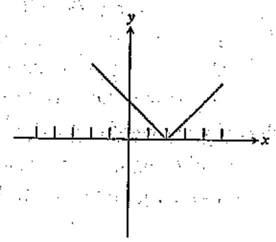
(g) $f(x) = |x + 2|$ (h) $f(x) = -\sin x$

(i) $f(x) = e^x - 1$ (j) $f(x) = 1 + \cos x$

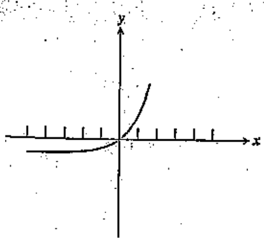
1.



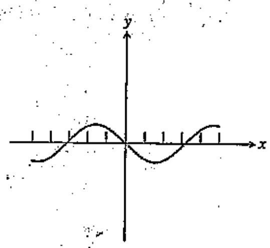
2.



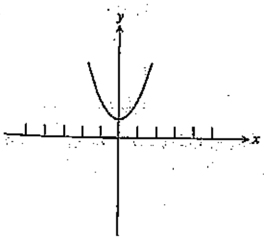
3.



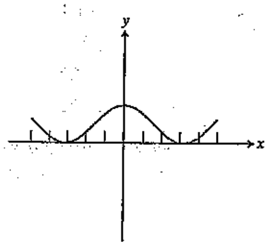
4.



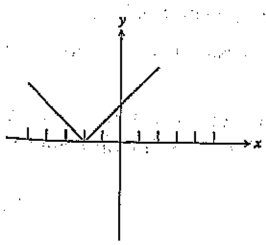
5.



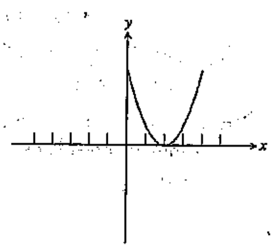
6.



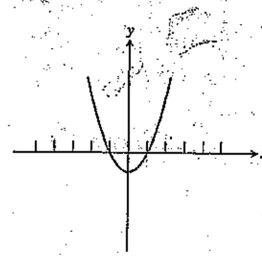
7.



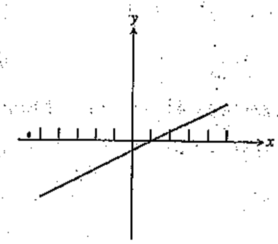
8.



9.



10.



In Exercises 11–18, find **(a)** the domain and **(b)** the range of the function.

11. $g(x) = x^3$ **12.** $f(x) = 35x - 602$

13. $g(x) = x^2 + 2x + 1$ **14.** $h(x) = (x - 2)^2 + 5$

15. $g(x) = 3|x| + 8$ **16.** $k(x) = \sqrt{4 - x^2} - 2$

17. $f(x) = \dfrac{x}{x^2 - 2x}$ **18.** $k(x) = \dfrac{1}{\sqrt{9 - x^2}}$

In Exercises 19 and 20, graph the function, and state whether the function is continuous at $x = 0$. If it is discontinuous, state whether the discontinuity is removable or nonremovable.

19. $f(x) = \dfrac{x^2 - 3}{x + 2}$ **20.** $k(x) = \begin{cases} 2x + 3 & \text{if } x > 0 \\ 3 - x^2 & \text{if } x \le 0 \end{cases}$

~~In Exercises 21–24, find all (a) vertical asymptotes and (b) horizontal asymptotes of the graph of the function. Be sure to state your answers as equations of lines.~~

[redacted] $\dfrac{5}{x^2 - 5x}$ [redacted] $\dfrac{3x}{x - 4}$

[redacted] $\dfrac{7x}{\sqrt{x^2 + 10}}$ [redacted] $\dfrac{|x|}{x + 1}$

In Exercises 25–28, graph the function and state the intervals on which the function is *increasing*.

25. $y = \dfrac{x^3}{6}$ **26.** $y = 2 + |x - 1|$

27. $y = \dfrac{x}{1 - x^2}$ **28.** $y = \dfrac{x^2 - 1}{x^2 - 4}$

In Exercises 29–32, graph the function and tell whether the function is bounded above, bounded below, or bounded.

29. $f(x) = x + \sin x$ **30.** $g(x) = \dfrac{6x}{x^2 + 1}$

31. $h(x) = 5 - e^x$ **32.** $k(x) = 1000 + \dfrac{x}{1000}$

In Exercises 33–36, use a grapher to find all **(a)** relative maximum values and **(b)** relative minimum values of the function. Also state the value of x at which each relative extremum occurs.

33. $y = (x + 1)^2 - 7$ **34.** $y = x^3 - 3x$

35. $y = \dfrac{x^2 + 4}{x^2 - 4}$ **36.** $y = \dfrac{4x}{x^2 + 4}$

In Exercises 37–40, graph the function and state whether the function is odd, even, or neither.

37. $y = 3x^2 - 4|x|$

38. $y = \sin x - x^3$

39. $y = \dfrac{x}{e^x}$

40. $y = x \cos(x)$

In Exercises 41–44, find a formula for $f^{-1}(x)$.

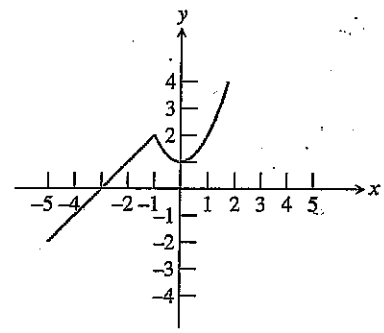
41. $f(x) = 2x + 3$

42. $f(x) = \sqrt[3]{x - 8}$

43. $f(x) = \dfrac{2}{x}$

44. $f(x) = \dfrac{6}{x + 4}$

Exercises 45–52 refer to the function $y = f(x)$ whose graph is given below.



45. Sketch the graph of $y = f(x) - 1$.

46. Sketch the graph of $y = f(x - 1)$.

47. Sketch the graph of $y = f(-x)$.

48. Sketch the graph of $y = -f(x)$.

49. Sketch a graph of the inverse relation.

50. Does the inverse relation define y as a function of x?

51. Sketch a graph of $y = f(|x|)$.

52. Define f algebraically as a piecewise function. [*Hint*: the pieces are translations of two of our "basic" functions.]

In Exercises 53–58, let $f(x) = \sqrt{x}$ and let $g(x) = x^2 - 4$.

53. Find an expression for $(f \circ g)(x)$ and give its domain.

54. Find an expression for $(g \circ f)(x)$ and give its domain.

55. Find an expression for $(fg)(x)$ and give its domain.

56. Find an expression for $\left(\dfrac{f}{g}\right)(x)$ and give its domain.

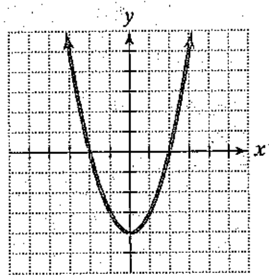
57. Describe the end behavior of the graph of $y = f(x)$.

58. Describe the end behavior of the graph of $y = f(g(x))$.

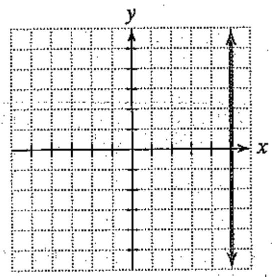
In Exercises 59–64, write the specified quantity as a function of the specified variable. Remember that drawing a picture will help.

~~**59. Square Inscribed in a Circle** A square of side s is inscribed in a circle. Write the area of the circle as a function of s.~~

7.

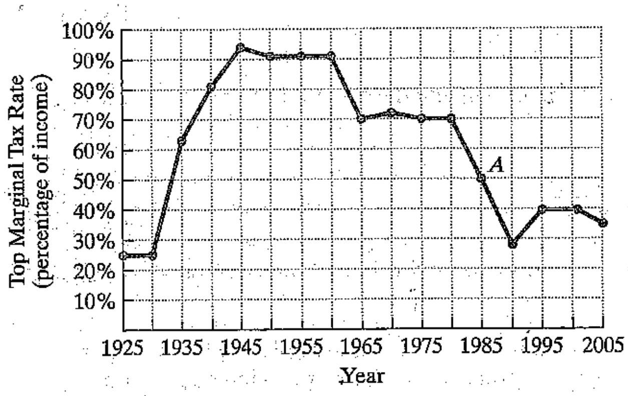


8.



The line graph shows the top marginal income tax rates in the United States from 1925 through 2005. Use the graph to solve Exercises 9–14.

Top United States Marginal Tax Rates, 1925–2005



Source: National Taxpayers Union

9. What are the coordinates of point A? What does this mean in terms of the information given by the graph?

10. Estimate the top marginal tax rate in 2005.

11. For the period shown, during which year did the United States have the highest marginal tax rate? Estimate, to the nearest percent, the tax rate for that year.

12. For the period from 1950 through 2005, during which year did the United States have the lowest marginal tax rate? Estimate, to the nearest percent, the tax rate for that year.

13. For the period shown, during which ten-year period did the top marginal tax rate remain constant? Estimate, to the nearest percent, the tax rate for that period.

14. For the period shown, during which five-year period did the top marginal tax rate increase most rapidly? Estimate, to the nearest percent, the increase in the top tax rate for that period.

1.2 and 1.3

In Exercises 15–17, determine whether each relation is a function. Give the domain and range for each relation.

15. $\{(2, 7), (3, 7), (5, 7)\}$

16. $\{(1, 10), (2, 500), (13, \pi)\}$

17. $\{(12, 13), (14, 15), (12, 19)\}$

In Exercises 18–20, determine whether each equation defines y as a function of x.

18. $2x + y = 8$

19. $3x^2 + y = 14$

20. $2x + y^2 = 6$

In Exercises 21–24, evaluate each function at the given values of the independent variable and simplify.

21. $f(x) = 5 - 7x$
   a. $f(4)$ b. $f(x + 3)$ c. $f(-x)$

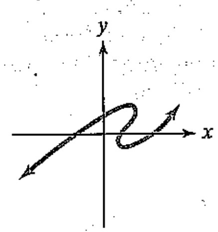
22. $g(x) = 3x^2 - 5x + 2$
   a. $g(0)$ b. $g(-2)$
   c. $g(x - 1)$ d. $g(-x)$

23. $g(x) = \begin{cases} \sqrt{x - 4} & \text{if } x \geq 4 \\ 4 - x & \text{if } x < 4 \end{cases}$
   a. $g(13)$ b. $g(0)$ c. $g(-3)$

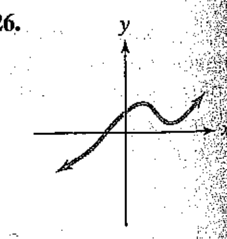
24. $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 12 & \text{if } x = 1 \end{cases}$
   a. $f(-2)$ b. $f(1)$ c. $f(2)$

In Exercises 25–30, use the vertical line test to identify graphs in which y is a function of x.

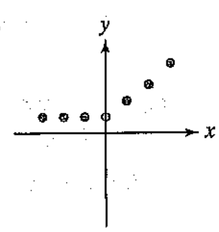
25.



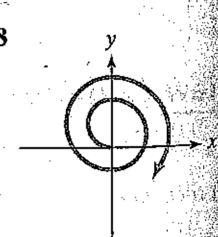
26.



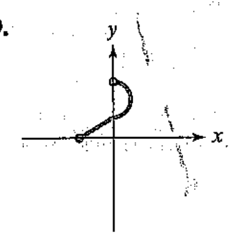
27.



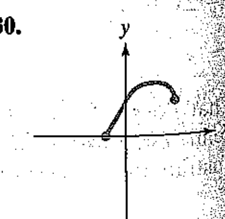
28



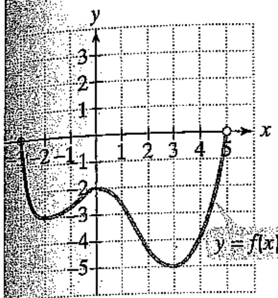
29.



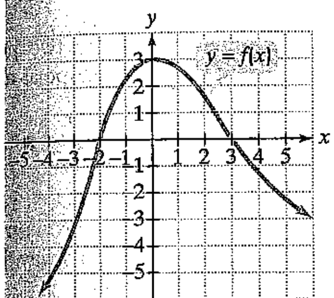
30.



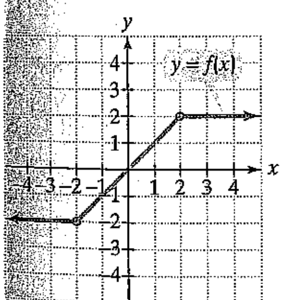
ercises 31–33, use the graph to determine **a.** *the function's ain;* **b.** *the function's range;* **c.** *the x-intercepts, if any;* **d.** *the ercept, if there is one;* **e.** *intervals on which the function is asing, decreasing, or constant; and* **f.** *the missing function s indicated by question marks, below each graph.*



$f(-2) = ?\quad f(3) = ?$



$f(-2) = ?\quad f(6) = ?$



$f(-9) = ?\quad f(14) = ?$

xercises 34–35, find each of the following:

a. *The numbers, if any, at which f has a relative maximum. What are these relative maxima?*

b. *The numbers, if any, at which f has a relative minimum. What are these relative minima?*

Use the graph in Exercise 31.

Use the graph in Exercise 32.

xercises 36–38, determine whether each function is even, odd, or her. State each function's symmetry. If you are using a graphing y, graph the function and verify its possible symmetry.

$f(x) = x^3 - 5x$

$f(x) = x^4 - 2x^2 + 1$

$f(x) = 2x\sqrt{1 - x^2}$

In Exercises 39–40, the domain of each piecewise function is $(-\infty, \infty)$.

a. *Graph each function.*

b. *Use the graph to determine the function's range.*

39. $f(x) = \begin{cases} 5 & \text{if } x \le -1 \\ -3 & \text{if } x > -1 \end{cases}$

40. $f(x) = \begin{cases} 2x & \text{if } x < 0 \\ -x & \text{if } x \ge 0 \end{cases}$

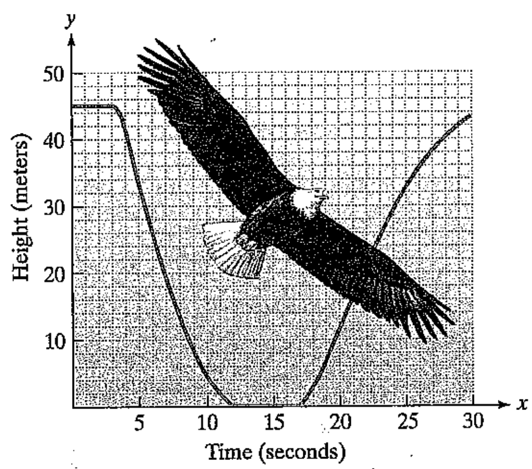
~~*In Exercises 41–42, find and simplify the difference quotient*~~

$$\cancel{\frac{f(x+h) - f(x)}{h}, \quad h \neq 0}$$

~~*for the given function.*~~

~~**41.** $f(x) = 8x - 11$~~ ~~**42.** $f(x) = -2x^2 + x + 10$~~

43. The graph shows the height, in meters, of an eagle in terms of its time, in seconds, in flight.



a. Is the eagle's height a function of time? Use the graph to explain why or why not.

b. On which interval is the function decreasing? Describe what this means in practical terms.

c. On which intervals is the function constant? What does this mean for each of these intervals?

d. On which interval is the function increasing? What does this mean?

44. A cargo service charges a flat fee of \$5 plus \$1.50 for each pound or fraction of a pound. Graph shipping cost, $C(x)$, in dollars, as a function of weight, x, in pounds, for $0 < x \le 5$.

1.4 and 1.5

In Exercises 45–48, find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

45. $(3, 2)$ and $(5, 1)$

46. $(-1, -2)$ and $(-3, -4)$

47. $\left(-3, \frac{1}{4}\right)$ and $\left(6, \frac{1}{4}\right)$

48. $(-2, 5)$ and $(-2, 10)$

In Exercises 49–52, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

49. Passing through $(-3, 2)$ with slope -6

50. Passing through $(1, 6)$ and $(-1, 2)$

51. Passing through $(4, -7)$ and parallel to the line whose equation is $3x + y - 9 = 0$

52. Passing through $(-3, 6)$ and perpendicular to the line whose equation is $y = \frac{1}{3}x + 4$

53. Write an equation in general form for the line passing through $(-12, -1)$ and perpendicular to the line whose equation is $6x - y - 4 = 0$.

In Exercises 54–57, give the slope and y-intercept of each line whose equation is given. Then graph the line.

54. $y = \frac{2}{5}x - 1$

55. $f(x) = -4x + 5$

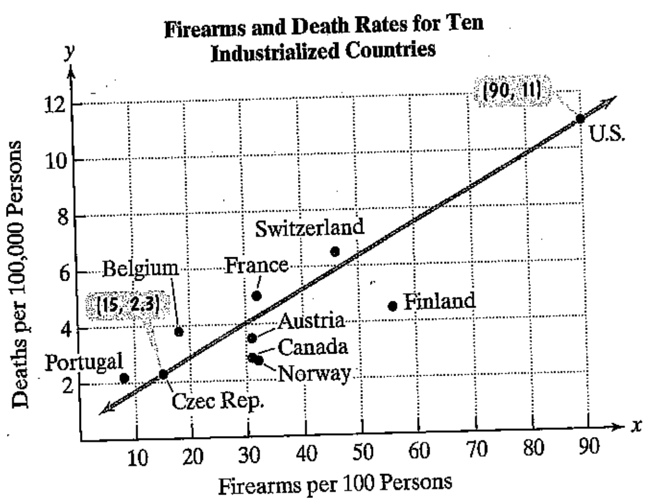
56. $2x + 3y + 6 = 0$

57. $2y - 8 = 0$

58. Graph using intercepts: $2x - 5y - 10 = 0$.

59. Graph: $2x - 10 = 0$.

~~The points in the scatter plot show the number of firearms per 100 persons and the number of deaths per 100,000 persons~~ for ~~industrialized countries with the highest death rate~~s.



Source: International Action Network on Small Arms

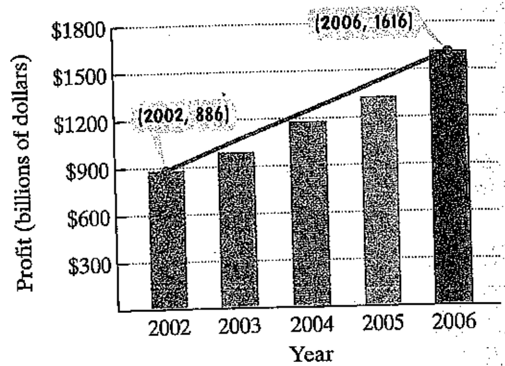
a. Use the two points whose coordinates are shown by the voice balloons to find an equation in point-slope form for the line that models deaths per 100,000 persons, y, as a function of firearms per 100 persons, x.

b. Write the equation in part (a) in slope-intercept form. Use function notation.

c. France has 32 firearms per 100 persons. Use the appropriate point in the scatter plot to estimate that country's deaths per 100,000 persons.

d. Use the function from part (b) to find the number of deaths per 100,000 persons for France. Round to one decimal place. Does the function underestimate or overestimate the deaths per 100,000 persons that you estimated in part (c)? How is this shown by the line in the scatter plot?

61. The bar graph shows the growth in corporate profits, in billions of dollars, from 2002 through 2006.



Source: Bureau of Economic Analysis

Find the slope of the line passing through the two points shown by the voice balloons. Then express the slope as a rate of change with the proper units attached.

62. Find the average rate of change of $f(x) = x^2 - 4x$ from $x_1 = 5$ to $x_2 = 9$.

63. A person standing on the roof of a building throws a ball directly upward. The ball misses the rooftop on its way down and eventually strikes the ground. The function

$$s(t) = -16t^2 + 64t + 80$$

describes the ball's height above the ground, $s(t)$, in feet, t seconds after it was thrown.

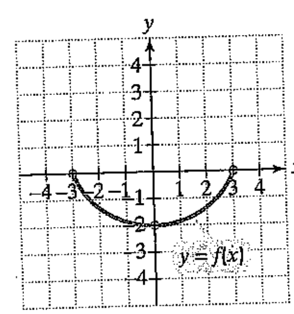
a. Find the ball's average velocity between the time it was thrown and 2 seconds later.

b. Find the ball's average velocity between 2 and 4 seconds after it was thrown.

c. What do the signs in your answers to parts (a) and (b) mean in terms of the direction of the ball's motion?

1.6

In Exercises 64–68, use the graph of $y = f(x)$ to graph each function g.



64. $g(x) = f(x + 2) + 3$

65. $g(x) = \frac{1}{2}f(x - 1)$

66. $g(x) = -f(2x)$

67. $g(x) = 2f\left(\frac{1}{2}x\right)$

68. $g(x) = -f(-x) - 1$

xercises 69–72, begin by graphing the standard quadratic ion, $f(x) = x^2$. Then use transformations of this graph to the given function.

$(x) = x^2 + 2$ **70.** $h(x) = (x + 2)^2$

$(x) = -(x + 1)^2$ **72.** $y(x) = \frac{1}{2}(x - 1)^2 + 1$

xercises 73–75, begin by graphing the square root function, $= \sqrt{x}$. Then use transformations of this graph to graph the function.

$g(x) = \sqrt{x + 3}$ **74.** $h(x) = \sqrt{3 - x}$

$(x) = 2\sqrt{x + 2}$

xercises 76–78, begin by graphing the absolute value function, $= |x|$. Then use transformations of this graph to graph the function.

$g(x) = |x + 2| - 3$ **77.** $h(x) = -|x - 1| + 1$

$(x) = \frac{1}{2}|x + 2|$

xercises 79–81, begin by graphing the standard cubic function, $= x^3$. Then use transformations of this graph to graph the function.

$g(x) = \frac{1}{2}(x - 1)^3$ **80.** $h(x) = -(x + 1)^3$

$(x) = \frac{1}{4}x^3 - 1$

Exercises 82–84, begin by graphing the cube root function, $= \sqrt[3]{x}$. Then use transformations of this graph to graph the function.

$g(x) = \sqrt[3]{x + 2} - 1$ **83.** $h(x) = -\sqrt[3]{2x}$

$(x) = -2\sqrt[3]{-x}$

Exercises 85–90, find the domain of each function.

$f(x) = x^2 + 6x - 3$ **86.** $g(x) = \dfrac{4}{x - 7}$

$h(x) = \sqrt{4 - x}$ **88.** $f(x) = \dfrac{x}{x^2 + 4x - 21}$

$g(x) = \dfrac{\sqrt{x - 2}}{x - 5}$

$f(x) = \sqrt{x - 1} + \sqrt{x + 5}$

Exercises 91–93, find $f + g, f - g, fg$, and $\frac{f}{g}$. Determine the main for each function.

$f(x) = 3x - 1, \quad g(x) = x - 5$

$f(x) = x^2 + x + 1, \quad g(x) = x^2 - 1$

$f(x) = \sqrt{x + 7}, \quad g(x) = \sqrt{x - 2}$

In Exercises 94–95, find **a.** $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(3)$.

94. $f(x) = x^2 + 3, \quad g(x) = 4x - 1$

95. $f(x) = \sqrt{x}, \quad g(x) = x + 1$

In Exercises 96–97, find **a.** $(f \circ g)(x)$; **b.** *the domain of* $(f \circ g)$.

96. $f(x) = \dfrac{x + 1}{x - 2}, \quad g(x) = \dfrac{1}{x}$

97. $f(x) = \sqrt{x - 1}, \quad g(x) = x + 3$

In Exercises 98–99, express the given function h as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$.

98. $h(x) = (x^2 + 2x - 1)^4$ **99.** $h(x) = \sqrt[3]{7x + 4}$

1.8

In Exercises 100–101, find $f(g(x))$ and $g(f(x))$ and determine whether each pair of functions f and g are inverses of each other.

100. $f(x) = \frac{3}{5}x + \frac{1}{2}$ and $g(x) = \frac{5}{3}x - 2$

101. $f(x) = 2 - 5x$ and $g(x) = \dfrac{2 - x}{5}$

The functions in Exercises 102–104 are all one-to-one. For each function,

a. *Find an equation for $f^{-1}(x)$, the inverse function.*

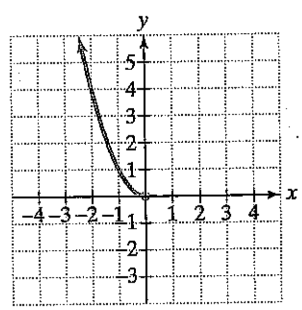
b. *Verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.*

102. $f(x) = 4x - 3$ **103.** $f(x) = 8x^3 + 1$

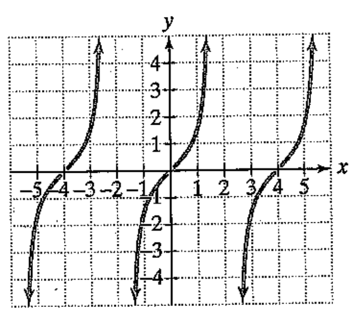
104. $f(x) = \dfrac{2}{x} + 5$

Which graphs in Exercises 105–108 represent functions that have inverse functions?

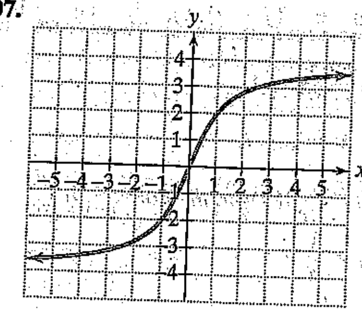
105.



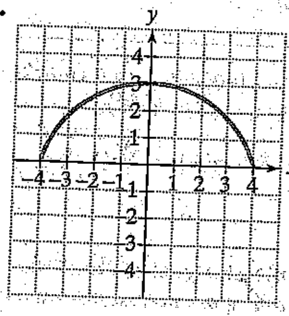
106.



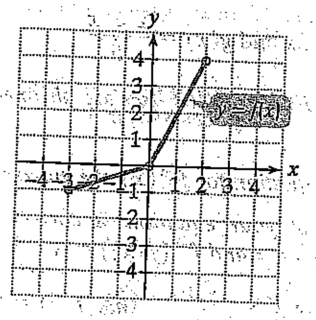
107.



108.



109. Use the graph of f in the figure shown to draw the graph of its inverse function.



In Exercises 110–111, find an equation for $f^{-1}(x)$. Then graph f and f^{-1} in the same rectangular coordinate system.

110. $f(x) = 1 - x^2, x \geq 0$ 111. $f(x) = \sqrt{x} + 1$

CHAPTER Test Prep VIDEOS

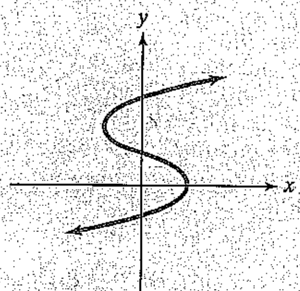
Chapter 1 Test

1. List by letter all relations that are not functions.

a. $\{(7,5),(8,5),(9,5)\}$

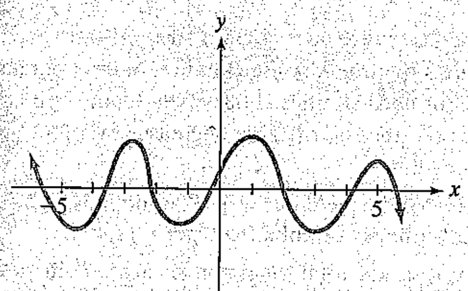
b. $\{(5,7),(5,8),(5,9)\}$

c.

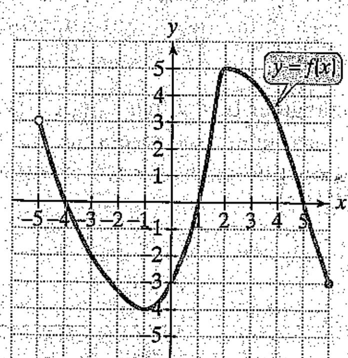


d. $x^2 + y^2 = 100$

e.



2. Use the graph of $y = f(x)$ to solve this exercise.



a. What is $f(4) - f(-3)$?

b. What is the domain of f?

c. What is the range of f?

d. On which interval or intervals is f increasing?

e. On which interval or intervals is f decreasing?

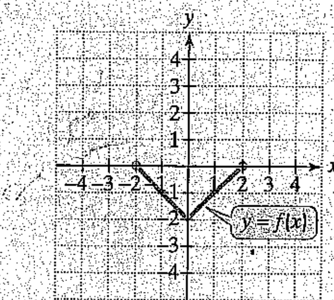
f. For what number does f have a relative maximum? What is the relative maximum?

g. For what number does f have a relative minimum? What is the relative minimum?

h. What are the x-intercepts?

i. What is the y-intercept?

3. Use the graph of $y = f(x)$ to solve this exercise.



a. What are the zeros of f?

b. Find the value(s) of x for which $f(x) = -1$.

c. Find the value(s) of x for which $f(x) = -2$.

d. Is f even, odd, or neither?

e. Does f have an inverse function?

f. Is $f(0)$ a relative maximum, a relative minimum, or neither?

g. Graph $g(x) = f(x+1) - 1$.

h. Graph $h(x) = \frac{1}{2}f\left(\frac{1}{2}x\right)$.

i. Graph $r(x) = -f(-x) + 1$.

j. Find the average rate of change of f from $x_1 = -2$ to $x_2 = 1$.

In Exercises 4–15, graph each equation in a rectangular coordinate system. If two functions are indicated, graph both in the same system. Then use your graphs to identify each relation's domain and range.

4. $x + y = 4$

5. $x^2 + y^2 = 4$

6. $f(x) = 4$

7. $f(x) = -\frac{1}{3}x + 2$

8. $(x+2)^2 + (y-1)^2 = 9$

9. $f(x) = \begin{cases} 2 & \text{if } x \le 0 \\ -1 & \text{if } x > 0 \end{cases}$

10. $x^2 + y^2 + 4x - 6y - 3 = 0$

11. $f(x) = |x|$ and $g(x) = \frac{1}{2}|x+1| - 2$

12. $f(x) = x^2$ and $g(x) = -(x-1)^2 + 4$

13. $f(x) = 2x - 4$ and f^{-1}

14. $f(x) = x^3 - 1$ and f^{-1}

15. $f(x) = x^2 - 1, x \ge 0$, and f^{-1}

In Exercises 16–23, let $f(x) = x^2 - x - 4$ and $g(x) = 2x - 6$.

16. Find $f(x-1)$.

17. Find $\dfrac{f(x+h) - f(x)}{h}$.

18. Find $(g - f)(x)$.

19. Find $\left(\dfrac{f}{g}\right)(x)$ and its domain.

20. Find $(f \circ g)(x)$.

21. Find $(g \circ f)(x)$.

22. Find $g(f(-1))$.

23. Find $f(-x)$. Is f even, odd, or neither?

In Exercises 24–25, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

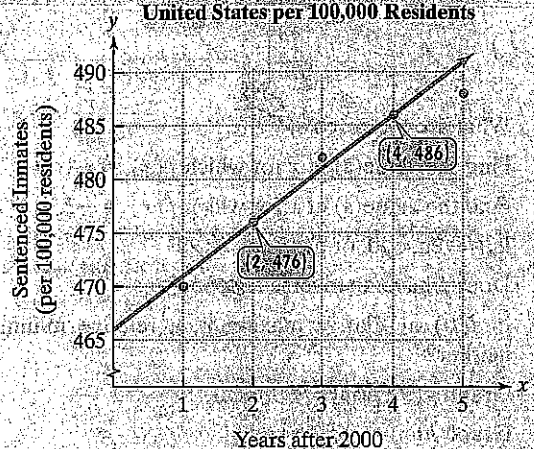
24. Passing through $(2, 1)$ and $(-1, -8)$

25. Passing through $(-4, 6)$ and perpendicular to the line whose equation is $y = -\frac{1}{4}x + 5$

26. Write an equation in general form for the line passing through $(-7, -10)$ and parallel to the line whose equation is $4x + 2y - 5 = 0$.

27. The scatter plot shows the number of sentenced inmates in the United States per 100,000 residents from 2001 through 2005. Also shown is a line that passes through or near the data points.

Number of Sentenced Inmates in the United States per 100,000 Residents



Source: U.S. Justice Department

a. Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the number of inmates per 100,000 residents, y, x years after 2000.

b. Write the equation from part (a) in slope-intercept form. Use function notation.

c. Use the linear function to predict the number of sentenced inmates in the United States per 100,000 residents in 2010.

28. Find the average rate of change of $f(x) = 3x^2 - 5$ from $x_1 = 6$ to $x_2 = 10$.

29. If $g(x) = \begin{cases} \sqrt{x-3} & \text{if } x \geq 3 \\ 3 - x & \text{if } x < 3 \end{cases}$, find $g(-1)$ and $g(7)$.

In Exercises 30–31, find the domain of each function.

30. $f(x) = \dfrac{3}{x+5} + \dfrac{7}{x-1}$

31. $f(x) = 3\sqrt{x+5} + 7\sqrt{x-1}$

32. If $f(x) = \dfrac{7}{x-4}$ and $g(x) = \dfrac{2}{x}$, find $(f \circ g)(x)$ and t
domain of $f \circ g$.

33. Express $h(x) = (2x + 3)^7$ as a composition of two functio
f and g so that $h(x) = (f \circ g)(x)$.